Dimensions of mathematical competence and its assessment in basic school

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Summary

Defining mathematical competence has been relevant since the 2000s (NCTM, 2000; OECD, 2000; Niss, 2003; Niss & Jensen, 2002). However, all the studies remain on the conceptual level and no analyses have established the empirically confirmed dimensions of mathematical competence. Describing the dimensions of mathematical competence is important in order to understand the development needs of students in the formation of comprehensive mathematical competence and to support teachers in choosing appropriate teaching methods.

One of the aims of this study is to describe the empirically distinguishable dimensions of mathematical competence based on the Mathematical Competency Research Framework (MCRF) developed in Sweden, which has been adapted to Estonian conditions (Johanson *et al.*, 2021) and to find suitable items for the assessment of the dimensions. The second aim of this study is to describe the level of mathematical competence of Estonian basic school students in the 3rd, 6th and 9th grades in different dimensions of mathematics competence and analyse how the levels change over time, indicating the focus on different dimensions of mathematical competence in the study process.

To approach these research questions, we rely on three tests for assessing mathematics competence created within the framework of the project DIGIVARA5 "DigiEfekt: The effect of the use of digital learning materials on learning and teaching on the example of Estonian basic education (1.05.2020–30.04.2023)": 1) the mathematical competence test for 3rd grades, 2) the mathematical competence test for 6th grades, 3) the mathematical competence test for 9th grades. All three tests are designed to assess mathematical competence through six dimensions (problem solving, reasoning, applying procedures, communication, building connections and representation) of mathematical competence. At least three tasks are provided in each test to measure each dimension (except for the tasks assessing the communication

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competence of the 6th and 9th grades). Data were collected from 13 different Estonian schools, including 21 3rd grades, 22 6th grades and 18 9th grades in two waves, including 345 different students from the 3rd grade, 312 different students from the 6th grade, and 254 different students from the 9th grade.

The designed tests for the assessment of mathematics competence enable the assessment of procedural competence, connections competence (starting from the 9th grade), representation competence, reasoning competence, problem solving competence and communication competence (except in the 9th grade). The IRT shows that the tests as a whole, the test elements and the evaluation scales are of good quality with item reliability of 0.99 and with student reliability of 0.83 except in the case of the 9th grade test with the reliability of 0.82 Factor models created to distinguish dimensions are characterised by good fit indicators, but correlations between factors are relatively strong which was expected because the literature also points out that the sub-competencies of mathematics competence are not strictly separated dimensions (Niss & Højgaard, 2019). Compared to the theoretical data model, the connection competence behaves the most unstable in the data of the 3rd and 6th grades, which may be due to the fact that school lessons and educational materials are structured on a topic-based basis (Ainevaldkond „Matemaatika". Põhikooli riiklik õppekava. Lisa 3, 2014) and making connections between mathematical concepts and objects is left to the students themselves to discover. By stressing connection competence more in the learning process, it would be conceivable to distinguish this dimension in real data. In the 9th grade, where it is possible to distinguish the connections competence dimension, the topic of making connections is introduced in the description of the objectives of the final exam in basic school mathematics (Põhikooli matemaatika lõpueksami eristuskiri, 2015).

The average result of mathematics competence over two measurements was 39% in the 3rd grade, 46% in the 6th grade and 44% in the 9th grade of the maximum points of the respective tests. Similarly, in all grade levels, the best results are in procedural competence (in the 3rd grade, 63%, in the 6th grade, 65% and the 9th grade, 60% of the maximum points of the respective dimension tasks) and in presentation competence (respectively 50%, 53% and 65% in the maximum points of the respective dimension tasks). The result was to be expected since procedural competence has received a lot of attention through national tests and exams, and the results are of the same magnitude as the results of the assessment obtained in the study. Based on the assessment of the dimensions of mathematics competence obtained in the study, it is necessary to pay more attention to the development of reasoning competence (results in the 3rd grade 15%, the 6th grade 33% and the 9th grade 29% of the maximum points) and the ability to express oneself correctly in mathematics, i.e.

mathematical communication competence (the result of the third and sixth grades are respectively 16% and 17% of the maximum points). Although the time difference between the two measurements carried out in the study was not large, it was possible to show statistically significant differences in the changes in the dimensions of mathematics competence. In the comparison of the first and second measurements, the data showed that there is a slowdown in the development of students' mathematical competence in the 6th grade. A statistically significant improvement in results occurred in four sub-competencies in the 3rd grade, in two sub-competencies in the sixth grade and in three sub-competencies in the 9th grade.

The next steps in the study are to improve the scales of the tests, to fit hierarchical data models of the dimensions of mathematical competence, and analyse what kind of background data affect the statistically significant changes in the mathematical competence dimension in this relatively short time period of 4 months.

Keywords: mathematical competence, general competence, assessment, item response theory, factor analysis