

A framework for analysing learning materials focusing on opportunities for developing mathematical competence: An application to an 8th-grade mathematics textbook

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Summary

In recent decades, mathematical competence has become an important concept in educational research and policy, highlighting the transferable skills essential to mathematical knowledge. The Estonian national curriculum aims to develop age-appropriate mathematical competence (Põhikooli riiklik õppekava, lisa 5, 2025). The textbook determines what and how mathematics is taught in classrooms, serving as a link between intended and implemented educational goals (Valverde et al., 2002). As Remillard (2005) notes, textbooks are instructionally neutral: depending on how they are used, the same book can either support competence development or devolve into procedural drill. Hence, a competency-based curriculum and educational goals do not necessarily guarantee competency-based teaching.

Research on mathematics textbooks is an established, methodologically diverse field, with several literature reviews published (Fan et al., 2013; Rezat, 2024; Rezat et al., 2021). Most studies emphasise content analysis (57%; Rezat, 2024; see also Fan et al., 2013), and many frameworks exist analysing the content of the mathematics textbooks (Gracin, 2018; Huntley et al., 2024; Li, 2000; O’Keeffe, 2013; Valverde et al., 2002). Such analyses are inherently linked to examining the learning opportunities provided by the materials; every framework targets a somewhat different research goal, e.g., topic-based analyses (Li, 2000), cross-national differences analyses (Valverde et al., 2002). Charalambous et al. (2010) conclude that no agreed approach has yet emerged for evaluating and comparing learning opportunities in textbooks.

Guided by the Estonian national curriculum’s aims, this study seeks to develop, on the basis of relevant literature and a synthesis of existing theoretical

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approaches, a framework for analysing mathematics learning materials that enables evaluation of whether these materials provide enough broad opportunities for the holistic development of mathematical competence. To achieve this, we formulated two research questions:

1. Which dimensions could an analytical framework for learning materials comprise to investigate the opportunities created within mathematics learning materials for the holistic development of students' mathematical competence?
2. What is the profile of tasks in one Estonian 8th-grade mathematics textbook regarding opportunities for developing students' mathematical competence overall and across different topics?

To select a framework suitable for the Estonian context and holistic competence aims, we reviewed the literature on textbook analysis and competence development. We adopted the mathematical competence research framework (Lithner et al., 2010) as a starting point, given its use in Estonia for national e-assessment analyses and competence measurement (Johanson et al., 2021a; Johanson et al., 2023). Based on an analysis of 225 tasks from the authors of the most popular mathematics textbooks in Estonia, we adapted the definitions of communication competence and interpretive activity to be suitable for the textbook context. Drawing on Pedaste et al.'s (2021b) definition of a problem task, we broadened the definition of problem-solving competence to align with a domain-competence-based approach. Following the studies by Jäder et al. (2020) and Lithner (2008), we replaced the activity "use" in the mathematics competence research framework with two activities: imitate (following a provided solution scheme with minimal adjustments) and construct (creating a scheme, algorithm, representation, or reasoning when the scheme is absent or inapplicable without substantive modification). Following Watson and Ohtani's (2015) research, we added a dimension that captures the potential for competence development – accounting for task structure, obligatoriness, support, and cognitive openness. The scale for this potential dimension is designed to measure both the substantive and didactic roles of a task that has the potential to develop competence, thereby shaping mathematical competence. Such a scale enables viewing the potential of tasks to foster each dimension of mathematical competence as a spectrum of learning opportunities differing in direction and scope.

To describe the profile of opportunities for developing mathematical competence in the textbook, we calculated distributions of the framework dimensions. The analysis is task-based. We examined the distribution for the original

chapters and the domains: algebra (1062 tasks) and geometry (985 tasks). We also highlighted the review tasks separately (totalling 623), as they have a structurally different content profile.

Although the framework for investigating mathematical competence in learning materials is three-dimensional, it enables profiling mathematics textbooks comparable to more dimensional international frameworks. The first dimension – mathematical sub-competences (6 categories) – covers cognitive demand (see Huntley et al., 2024; Li, 2000), expected skills (Valverde et al., 2002), expectations (O’Keeffe, 2013), and partially the context dimension (Huntley et al., 2024), as well as Li’s (2000) view of mathematicalness and, to some extent, Gracin’s (2018) activity dimension. The second dimension of the framework – activities (4 categories) – partially reflects Gracin’s (2018) activities and levels of complexity and Valverde et al.’s (2002) activity structures for expected skills, but offers a more detailed perspective by distinguishing between imitation and construction as separate activities and is universal for all subcompetences. The third dimension – potential for competence development (five categories) – is linked to theories of learning opportunities and developmental levels (Bingolbali & Bingolbali, 2019; Hadar, 2017; Vygotsky, 1978) as well as studies on the impact of task structure (Valverde et al., 2002; Watson & Ohtani, 2015). Thus, the framework consolidates and extends existing approaches within a three-dimensional perspective.

The analysis of an Estonian Grade 8 mathematics textbook reveals uneven coverage of competence dimensions, a trend that also characterises competence-level assessments (Johanson et al., 2023). Procedural tasks dominate (78.4%; cf. 79% in U.S., 80% in Singapore, 63% in Turkey; Özer & Sezer, 2014), of all tasks 59% focus on imitating solution procedures provided in the learning material; according to Valverde et al. (2002), only 8% of textbooks aimed at 13-year-old students have 50% or more routine procedural tasks (comparable with imitation in our case). Approximately 5% of tasks in this study require reasoning competence (whereas Valverde et al. (2002) reported that 23% of international textbooks include mathematical reasoning within a range of 0–10% of all tasks), less than 2% require evaluation, and less than 4% require interpretation.

The textbook plays a central role in classroom practice (Lepik et al., 2015; Valverde et al., 2002), and, as such requires supplementary materials that support the development of mathematical language and reasoning skills, particularly in culturally diverse and differentiation-demanding classrooms. An increasing number of tasks requiring reasoning and evaluation would better align with the goals of the Estonian national curriculum for basic school, while the rise of artificial intelligence stresses, in addition, the importance of conceptual understanding and interpretation skills.

Based on the findings of this study, we propose the following general recommendations. Textbooks should indicate the mathematical competence dimensions addressed by each task, enabling teachers to monitor the representation of all dimensions in classroom practice. The proportion of underrepresented dimensions of mathematical competence should be increased to ensure that teachers have options within these categories as well. The overall profile of competence-related activities should be more balanced but not necessarily explicitly marked. Interpretation activities should focus on recognising and understanding connections. Tasks with restrictive potential, intended to foster mathematical creativity, should be highlighted with a separate marker. In the case of algebra, a clear pedagogical decision should be made regarding whether the acquisition of formulas and methods is supported. If so, the number of tasks with supported potential should be increased, reducing the number of drill-oriented tasks.

Keywords: mathematical competence, domain-specific competence, mathematics textbook analysis, quantitative analysis, content analysis.